Chapter 5
The Perception of Spatial Depth in Kepler's and Descartes' Optics: A Study of an Epistemological Reversal

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Abstract This paper is devoted to the explanation of the location and distance of objects in three-dimensional space through vision in the work of two major opticians of the 17th century, namely Kepler and Descartes. I show that, in his *Dioptrique*, Descartes took up from Kepler's *Ad Vitellionem Paralipomena* a psychological procedure involved in vision and consisting in a trigonometric operation. But, whereas Kepler had resorted to this procedure to account for the illusory, imaginary location of objects seen through reflection or refraction, Descartes applied it to the perception of distance in non-deceptive direct vision. This brings about a complete shift regarding the epistemological value of the psychological operations involved in vision. I indeed show that this displacement reveals that Descartes saw his natural geometry of vision as an epistemological foundation for the integration of sense perception into his physics.

Optics in the narrow sense of the word first includes a geometrical conception of space, since a certain kind of space is implied by theories relying on rays propagating rectilinearly from one point to another, or from one object to another, or being reflected and refracted. This is Euclidean, geometrical space, in which it is possible to draw rectilinear lines corresponding to the passage of the visual ray (and later on of the light ray) outside the eye. But if we consider optics from a historical perspective, we realize that it also implies a psychological treatment of spatial data, which gives rise to space as perceived by the observer through the sense of sight: how do we perceive things with the dimension of depth? Belonging to a more psychological realm, this conception of space is linked to the capacity of our eyesight to give perception of spatial depth. When opening our eyes, we do not only see patches of colored shapes in two dimensions, but objects in a three-dimensional space. Through vision, we are able to distinguish different objects and their spatial

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relationships. But how can what is purely external to our body and soul be seen in our thought as it is in extended space? From antiquity to the 17th century, this perceived dimensionality of space was not conceived as something purely external to extended space through which rays travel. I want to question, from a historical point of view, how these two ways of conceiving visual space are articulated. To that end, I intend to concentrate on two major figures of early modern optics, namely Kepler and Descartes. Theoretical connections between the approaches adopted by both authors have already been noticed in the available scholarship.[1] And they are mainly substantiated by the fact that Descartes himself declared, in a letter to Mersenne, that Kepler was his "first master in optics."[2] But the studies that emphasize this link have concentrated on what appears more "modern" in the optics of the two scientists, that is to say the analysis of the eye as an optical instrument and the function ascribed to the retinal picture in the process of vision. However, other studies have also emphasized that Kepler's theories remained much more embedded in an older framework.[3] Unlike Kepler, Descartes, as far as optics is concerned, is often seen as belonging to the modern part of science.[4] It might be due to his one and only success in demonstrating a law that is valid for contemporary optics, the law of refraction. It might also be due to his mechanization of light, which breaks away from the Scholastic as well as the Neo-Platonic conceptions of light. Or it could come from the mechanization of the physiological processes involved in vision going from the retina to the optical nerve, the chiasma and the brain. But as far as Kepler is concerned, things appear much different in the historiography. Some, like David Lindberg, consider Kepler as belonging to the tradition of perspectivist optics (beginning with Roger Bacon in the 13th century and based on the optics of Ibn Al-Haytham or Alhazen).[5] Others, like Stephen Straker or Raz Chen-Morris and Ofer Gal, insist on the very modern dimensions of his optics centered on the sole study of the rectilinear propagation of light rays (and thereby rejecting theories of *species sensibiles*).[6] What has rarely been noticed is that the picture becomes more complicated when one notices that Descartes also takes up another major dimension of Kepler's optics that is precisely not the one concerned with the study of the propagation of light according to optical principles inside the eye, but is rather concerned with a kind of psychological procedure to determine the distance at which objects seen are evaluated by eyesight.

[1]See for example Simon (1997).

[2]Descartes to Mersenne, 31 March 1638, AT II 86.

[3]See for example Lindberg (1976, 1986).

[4]See for example Simon (1975).

[5]See Lindberg (1976, 86, 122, 205, 207–208; 1978, 354). For a similar position, see Buchdahl (1972).

[6]See Straker (1971, vi–vii and *passim*; 1976); Gal and Chen-Morris (2010). A. Mark Smith proposes a more balanced interpretation: see Smith (1998, 13, 39, 42).

In order to account for this seemingly unexpected connection and to explain its significance, this paper will deal with two apparently different but ultimately intertwined, issues: (1) one is that of the evaluation of the distance of objects perceived by sight; (2) the other is that of the location of a reflected or refracted image. I shall simultaneously treat these two issues throughout this paper because I want to show that Descartes dealt with the first one in the way that Kepler dealt with the second one. Therefore the *terminus ad quem* of this paper is mainly the first issue as it is dealt with by Descartes. But to understand the deep epistemological reversal Descartes effected with regard to the Keplerian procedure, I will have to locate the latter procedure in its historical background and give insight on the status of reflected and refracted images and their location in visual space in the optical tradition from Euclid to the perspectivists. This will enable me to show that, while Kepler introduced a psychological procedure to account for the perception of what he considered to be errors concerning reflected or refracted images, Descartes took up this very procedure and transferred it to non-deceptive direct vision.

5.1 Perceiving Distance and Spatial Properties in Optics Before Kepler

It would be an anachronism to think that optics, since antiquity, has been a modern discipline insofar as it apprehended the external world only by means of mathematics. Indeed, from its very beginning, optics had never been independent from philosophical considerations that have had considerable consequences on its epistemology. This is mainly due to the fact that optics, at first, did not deal only with light but with the visible.[7] The fact that "optics" (in Greek *optica*) is derived from *ops* (the eye) clearly suggests that the reference-point of the discipline was the eye, not light. But since the eye, as the percipient organ, is not in direct contact with the objects seen, optics had to account for what could be called "the problem of vision": namely the fact that we do not see distant objects as being in our mind or in our eye but in the extended external world. The perception of visual space was therefore at the core of its interests.

For Euclid, this problem was solved by simply assuming that visual rays issue from the eye, forming a cone whose base is the object seen and whose vertex is the eye. Perception of distance was then just a question of interpreting line segments according to their apparent magnitude.[8] And this depended, for Euclid, on the

[7]This is one of the main theses defended by Gérard Simon: see Simon (1988).

[8]See Euclid (1945, def. 1–2, 4, 357).

magnitude of the visual angle (that is to say, the aperture of the visual cone). As a consequence, equal lines that are located at different distances do not appear as being the same size. Euclid's theory consisted of interpreting the appearance of real spatial configurations in a geometrical perspective.[9]

Ptolemy took up the theory of the visual ray and cone and directly applied it to the perception of distance, as sensed by the length of the visual rays. But, contrary to Euclid, Ptolemy did not limit himself to the perspective interpretation of apparent magnitudes and shapes and tried to account for the perception of three-dimensional objects.[10] The perception of the location and distance of objects in Ptolemy's *Optics* mainly relies on the capacity the visual rays have, within a continuous visual cone, to sense their own length.[11] This is an important difference with Euclid's optics in which only the aperture of the visual angle was taken into consideration to determine the size and distance of objects. Ptolemy's more realistic account of three-dimensional properties of bodies suggests that he was aware that the visual perception of space did not only consist in a perspective representation that one would have to account for afterwards, in terms of objective spatial properties, but that it was, right from the start, an interpretation of visual elements in three dimensions. One does not find a whole psychology of vision in Ptolemy,[12] but one should note that it is not really required since the visual ray, insofar as it is a mix of psychic and spatial reality, "inhabits," as it were, the external world.[13] The evaluation of the depth of the visual space ultimately relied on this projection of the soul into spatial reality (for example, this is required to understand how, for Euclid, the percipient can have knowledge of the width of the visual cone corresponding to the visual angle). Therefore, according to Gérard Simon, the question of the visual perception of space was not a real problem for Ptolemy's optics and it only became a problem in the Middle Ages.[14]

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[9]Euclid (1945, 357): "Of equal spaces located upon the same straight line, those seen from a greater distance appear shorter"; Euclid (1945, 358): "Objects of equal size unequally distant appear unequal and the one lying the nearer to the eye always appear larger".

[10]On the visual perception of space (including location, size, shape, and motion) in Euclid's and Ptolemy's optics, see Lejeune (1948, 85–123).

[11]Ptolemy (1996, II 26, 81–82): "The visual faculty also discerns the place of bodies and apprehends it by reference to the location of its own source-points [i.e. the vertices of the visual cones]…as well as by the arrangements of the visual ray falling from the eye upon those bodies. That is longitudinal distance [is determined] by how far the rays extend outward from the vertex of the cone…whatever is seen with a longer ray appears farther away…".

[12]On the psychological elements present in Ptolemy's *Optics* to perceive the size, shape, location, movement and other properties of objects, see Smith (1988).

[13]See Simon (1994).

[14]See Simon (1981, 311).

Indeed when, with Alhazen's optics, the visual ray was replaced by the light ray between the object seen and the eye[15]; the former solutions based on visual rays devised to account for the sense of depth and distance in vision could no longer stand.[16] Alhazen's doctrine was known in western Christendom through the Latin translation of his optical work under the title *De aspectibus*. It was integrated by Roger Bacon, John Pecham and Witelo within the so-called perspectivist tradition which was to be the basis of Kepler's examination of optical matters.[17]

As opposed to the visual ray, the light ray could not possess self-knowledge of its own length. With the visual ray, emission theory endowed the eye with sensitivity of direction and depth: the object could be located in space thanks to the direction and length of the visual ray terminating on it. By rejecting emission theory, Alhazen had to do without the directional sensitivity it involved: the crystalline lens could only sense the presence of light, but no indication was given regarding where it came from.[18] Moreover, since light diffuses in every direction, the eye received on each of its parts overlapping light rays coming from various points on the object seen. It was thus hardly understandable how different parts of the perceptual space could ever be distinguished through this resulting confusion. However, Alhazen considered that the visible form of the object was impressed on the crystalline lens through the selection of light rays that were perpendicular to the surface of the eye and therefore not refracted and were the only ones to be sensed.[19] This allowed a one-to-one correspondence between the form sensed in

[15]Simon (1992, 213): "L'enjeu est de remplacer, par la propagation rectiligne *de la lumière*, l'antique rectilinéarité prêtée aux rayons *visuels* issus de l'œil." ("What is at stake is to replace, through the rectilinear propagation *of light*, the ancient rectilinearity that was attributed to the *visual* rays coming from the eye.") Lindberg (1967, 323): "*every visible object* is seen by the emission of its own light…Light and color remain the primary visible intentions, and the others are perceived through their mediation." Alhazen (2001, I 5, Sect. 27, vol. 2, 347): "the form of the visible object that sight perceives depends entirely upon the light possessed by that visible object, as well as upon the light that shines upon the eyes when that visible object is perceived, and upon [the light that illuminates] the aerial medium between the eyes and the visible object." Alhazen (2001, I 7, Sect. 1, vol. 2, 355): "it is a property of light to affect sight, whereas it is in the nature of sight to be affected by light." Alhazen (2001, I 7, Sect. 6, vol. 2, 356): "sight senses the light and colors that are in the surface of the visible object and…they pass through the transparency of the tunics of the eye." Alhazen, who would be followed by Witelo, clearly rejected visual rays: see Alhazen (2001, II 3, Sect. 71, vol. 2, 449); Witelo (1991, III 5, 111–112). Even if Roger Bacon and John Pecham would follow Alhazen on most topics, they would nevertheless maintain the hypothesis of visual rays as complementary to light rays in the process of vision. See Pecham (1970, I 67, 143); Lindberg (1967, 326, 339–341).

[16]As A. Mark Smith rightly notes, "the visual perception of the spatial characteristics of things is not immediate in the way our tactile perception of those things seems to be. The ulterior implication of the arguments is that there is absolutely nothing intuitive about spatial perception; it is an entirely inferential process" (Alhazen 2001, vol. 2, 543n.81).

[17]See Lindberg (1976).

[18]See Lindberg (1967).

[19]However, one must acknowledge that Alhazen and the perspectivists also tried to take into account to some extent the peripheral rays in the process of vision. The focus on the selection of the perpendicular rays comes largely from Kepler's criticism.

the crystalline lens and the object seen. At any rate, and even if the selective sensitivity of the crystalline lens prevented any optical confusion, the eye received only the forms of light and color, not the form of distance or of location. This means that Alhazen had to explain how three-dimensional vision was possible from this set of colored and luminous patches deprived of any spatial depth.[20] On the purely optical level, Alhazen considered images coming from each point on the object, (*optical forms* radiated in every direction from every point on the object) rather than a replica of the whole object.[21] As a consequence, the psychological process is meant to bridge the gap between a punctiform, purely geometrical analysis of the visual space (which can be analyzed into points) and a more "phenomenological" conception of space filled with objects distinct from one another and set in different positions relative to each other (and apprehended through a global *resultant psychological form* of the object).[22]

From the eye to the brain there is a continuous transmission of forms from the crystalline to the optical nerves, to the imagination, until the *ultimum sentiens* (last sentient). Then the faculty of judgment (*virtus distinctiva*) distinguishes and compares the different colors, the similarity or difference, and the relations between different parts (what Alhazen calls *intentiones visibiles*, including distance).[23] The outcome of this process is the production of a clear perception of the composition of the whole object. This results in the conception of an implicit judgment based on processes of recognition, differentiation, and deduction. But these processes

[20]However, note that the form received by the crystalline was not a real picture, as Kepler's retinal picture would be, but a purely sensorial form.

[21]Admittedly, Alhazen considered that the "form" of light and color was transmitted to the eye. But this word is not to be understood as a strict equivalent of the *species* which will be introduced in perspectivist optics only by Roger Bacon and Witelo. Neither can the form of light be strictly assimilated to the Aristotelian form which transports the appearance of the whole object. Indeed, even if Alhazen did not define the term in his *Optics*, the form of light emanates from point-sources and not from the visible object as a whole. See Sabra (1989, 116): "he proposed to subject form to geometrical analysis, something which no Aristotelian before him had thought of doing…" This form is only related to light and illuminated color, and not to the other properties, be they also visible like shape, of the object. Sabra (1989, 119): "Form in this sense is not to be confused with visible shape or figure or appearance; it simply refers to the light and colour themselves as physical properties of the luminous and coloured object." See also Smith (1981, 587) (for a different interpretation which associates Alhazen's form to Aristotle's, see Lindberg (1967, 335; 1976, 78–79). The selection of perpendicular rays by the crystalline then gives rise to an optical form which is an optical array, a pattern of light and colors from which the form of the object will be perceived (see Sabra 1978, 169, 173). One can therefore distinguish at least between three levels of form: the punctual form of light and color radiating from the object to the eye, the optical form sensed by the crystalline and the form of the object as it is perceived by the process of visual perception (for a similar analysis see Sabra 1989, 129–131). The form of light is thus expressed by the word *ṣūra* in Arabic (translated by *forma* in Latin, but never by *species*), whereas light as considered as an intention (that is to say as discerned by judgment) is referred to by the word *ma'nā*. See Sabra (1989, 117n.4).

[22]See El-Bizri (2005).

[23]On the perception of distance, see Alhazen (2001, II 3, Sects. 67–93, vol. 2, 448–457).

occur so quickly and are so customary that they are rarely ever noticed as such.[24] The intentions of distance and location are not transferred to the percipient by the forms of light and colors received in the crystalline and therefore must be elaborated by a kind of reasoning. The evaluation of distance (that is to say of the extent of space between observer and object[25]) by sight relies on an evaluation of the size of objects and intermediary spaces that are in a more or less continuous and ordered range between the percipient and the remote object.[26] The perspectivists took up this way of conceiving the evaluation of distance by sight. Roger Bacon included it among the things that can be perceived by means of syllogism.[27] Pecham emphasized that "the distance [from the observer] to the visible object is not perceived by sight, but is determined by reasoning [*ratiocinatione*]…"[28] Witelo kept Euclid's evaluation by the magnitude of the visual angle[29] but added the assistance of a *virtus distinctiva* inherited from Alhazen.[30]

5.2 The Status of Reflected and Refracted Images in Optics Before Kepler

I now turn to the second topic of inquiry of this paper that will ultimately return to the first, namely that of the evaluation of the location and distance of an object seen by reflection or refraction. For Ptolemy, it was not enough to account for vision as it enabled us to see objects as they were. But a good deal of the optics of the time was also meant to account for perceptual illusions and errors. Given the fact that visual rays were conceived as the externalization of our sense organs, it became indeed intriguing that we sometimes did not see objects where they were or in their true shape or colors. The study of reflected and refracted images was included in this analysis of perceptual errors, insofar as they implied that the

[24]See Alhazen (2001, II 3, Sects.16–42, vol. 2, 431–438). See Sabra (1978).

[25]See Alhazen (2001, II 3, Sect. 68, 448).

[26]See Alhazen (2001, II 3, Sects. 72–77, vol. 2, 449–452). Alhazen (2001, II 3, Sect. 80, vol. 2, 453): "sight does not perceive the magnitude of the distance of a visible object unless its distance is spanned by a continuous, ordered range of bodies, and unless sight perceives those bodies and determines their measures accurately".

[27]See Bacon (1996, II 3.3, 207–213). Bacon notes that "distance can be grasped and certified if it is moderate, through the continuity of sensible bodies intervening between the eye and the distant object." (207) He adds that "distance is grasped…when a sequence of bodies is arranged continuously between the eye and the object, provided that the distance is moderate and that the eye will have inspected those bodies and certified their magnitudes" (211).

[28]Pecham (1970, I 63, 141). Proposition 64 is entitled "The magnitude of a distance is certified by the resolution of the intervening space into magnitudes of exactly known measure" (Pecham 1970, I 64, 141).

[29]See Risner (1572, Witelo IV 7, 120).

[30]See Risner (1572, Witelo, IV 9–10, 121–122).

visual ray was broken and thus impeded in seeing the object where it was. That means that reflection and refraction were not mainly conceived as natural phenomena produced by physical laws, but primarily as errors of sense perception where the percipient was misguided in identifying the object in the wrong place (for example in the case of an image reflected by a mirror or refracted in a transparent medium) or with a wrong shape or wrong color (when the object was seen through a colored glass for example). However, such illusions were considered to obey some principles that could be made explicit by optics. This led Ptolemy,[31] in the case of images reflected in a mirror, to formulate the *cathetus* rule: the object whose image is reflected in a mirror is seen at the intersection of (1) the visual ray drawn from the observer's eye to the point of reflection in the mirror and (2) the perpendicular dropped from the object to the mirror.[32] This rule initially based on visual rays was adopted by Alhazen[33] and taken up by the perspectivists Roger Bacon,[34] John Pecham,[35] and Witelo,[36] even if they considered that vision occurred exclusively or mainly by way of intromission. Pecham considered that this was rendered possible by the fact that "the length of rays are perceived by the eye,"[37] thus returning to Ptolemy's optics. Bacon, in his *Perspectiva*, was even more explicit, stating that "vision occurs by extramission."[38]

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[31] This rule was also formulated in the pseudo-Euclid's *Catoptrics*.

[32] Ptolemy (1996, III 3, 131): "The first of these principles asserts that objects seen in mirrors appear along the extension of the [incident] visual ray that reaches them through reflection, the resulting line-of-sight being determined by the placement of the pupil with respect to the mirror. The second principle asserts that particular spots [on a visible object] seen in mirrors appear on the perpendicular dropped from the visible object to the mirror's surface and passing through it [i.e., the cathetus rule of reflection]." The third principle states the equality of the angles of incidence and of reflection.

[33] See Alhazen (2006, V 2, vol. 2, 385): "The image-location of any point is the point where the line of reflection intersects the normal imagined [to extend] from a point on a visible object to the line tangent to the common section of the surface of the mirror and the plane of reflection, or [to the common section] of the plane that coincides with [the plane of the] mirror and the plane of reflection".

[34] See Bacon (1996, III 1.2, 261–263).

[35] See Pecham (1970, II 19–20, 169–173).

[36] See Witelo (1983, V 37, 120–122). This proposition is entitled "The locus of the image of an object seen in any mirror must lie on the point of intersection of the line of reflection and the cathetus of incidence".

[37] Pecham (1970, II 20, 171).

[38] Bacon (1996, 261–263): "Along with these matters, it must be known that an object seen by reflection does not appear in its true place, because sight is accustomed to seeing by means of straight lines and [to judging] visible things to be at the extremities of these lines, and therefore it does not perceive the bending that occurs in reflection. Consequently, it judges the visible object always to be on [the rectilinear extension of] the visual ray, and the place of the image, which we call the 'appearance of the object', to be at one of its points. This is possible because vision occurs by extramission, and therefore vision judges the object to be in the direction of the species issuing from the eye. Although the object does not always appear in the same place, in most cases it appears at the intersection of the visual ray and the cathetus (the perpendicular drawn from the visible object to the mirror)".

5.3 The Perception of the Location and Distance of Reflected and Refracted Images in Kepler's Optics: An Approach Between Physics and Psychology

The historical background for the perception of distance and for the location of reflected images that I have presented will enable me to show first how Kepler's criticism of perspectivist optics on the issue of reflection remained nonetheless largely dependent on a number of conceptions that were to be found in the works of perspectivist opticians, and second how Descartes, by appropriating Kepler's analysis of the location of reflected and refracted images ended by transferring it to the problem-context of distance perception, thereby producing an important epistemological reversal.

In the 17th century, Kepler and Descartes completely reinvented optics and gave it a new foundation. The perception of spatial depth was affected by this reinvention. Now, Kepler and Descartes each appealed to a procedure of triangulation in order to account for the distance of what is seen. But Kepler did not apply the process of triangulation to exactly the same object as Descartes. It is therefore crucial to identify the functions the two authors gave to that process in their optics and in their theories of vision. Specifically, the idea of the evaluation of distance by a psychological process seems a bit alien to a modern optics centered on an analysis of light propagation. As we have seen, this had been a common approach in optics at least since Ptolemy and the solutions that were formulated relied heavily on various conceptions of the process of vision. Thus, what can Kepler's and Descartes' approaches teach us?

In his *Ad Vitellionem Paralipomena quibus Astronomiae Pars Optica traditur*, Kepler refuted the perspectivist theory of vision, in particular the privilege given to perpendicular light rays in the transmission of visible forms.[39] Kepler wanted to show that all light rays were transmitted through the eye but were refracted in such a way as to produce a punctiform picture of what is seen on the retina.[40] All along until the retina, light rays remain light rays and are not coupled with a parallel form that could be assimilated by the soul. As a consequence, through his new theory of light transmission, Kepler conducted a reform of optics that induced a new definition of its boundaries. Optics became mainly a science of light.[41] But the study of light rays could go no further than the opaque screen of the retina, after which the rays could no longer travel in a transparent media.[42] As a consequence,

[39]See Kepler (2000, V 4, 220) (KGW II 183).

[40]For a presentation of the demonstration of the formation of the retinal picture from light rays by Kepler, see Lindberg (1976, 193–202).

[41]See his preface to the *Dioptrice*, KGW IV 340, 22–23.

[42]See Kepler (2000, V 2, 180) (KGW II 152).

it seems impossible, according to Kepler's requirements, to explain how vision, in the mental sense of the term, occurs from the impression of the reversed and inversed picture of the outer world on the retina. This belongs to the work of "*Physici*" (natural philosophers).[43] It is thus with an analogical relation, expressed in a very elliptic way, between the retinal picture and the vision of the external world that Kepler formulated a potential explanation which eventually amounted to a refusal to enter into any explanatory process: "*Nam ut pictura, ita visio*"[44] However, we do not see the external world as if it were a painting. Except in the case of a perfectly made trompe-l'œil, we can always distinguish between the real spatial depth of the external world and the flatness of a painting. Consequently, by refusing to account for vision on the basis of the retinal picture, Kepler seemed to deprive optics of its psychological dimension. For, contrary to what the perspectivist theories proposed, what was transmitted through the eye onto the retina remained a purely luminous entity and was not coupled with a form of the visible realm that could be assimilated by the mind.

Now, despite this seeming exclusion of psychological considerations in optics, in the third chapter of his *Ad Vitellionem Paralipomena* dealing with reflected and refracted images, Kepler introduced a type of optical entity, *imago*, distinct from *pictura*,[45] and involving a clear psychological dimension. *Imago* was defined as follows:

> The Optical writers say it is an image [*imaginem*], when the object itself is indeed perceived along with its colors and the parts of its figure, but in a position not its own, and occasionally endowed with quantities not its own, and with an inappropriate ratio of parts of its figure. Briefly an image is the vision of some object conjoined with an error of the faculties contributing to the sense of vision. Thus, the image is practically nothing in itself, and should rather be called imagination [*imaginatio*]. The object is composed of the real form of color and light and of intensional quantities.[46]

[43]Kepler (2000, V 2, 180): "How this image or picture is joined together with the visual spirits that reside in the retina and in the nerve, and whether it is arraigned within by the spirits into the caverns of the cerebrum to the tribunal of the soul or of the visual faculty; whether the visual faculty, like a magistrate given by the soul, descending from the headquarters of the cerebrum outside to the visual nerve itself and the retina, as to lower courts, might go forth to meet this image—this, I say, I leave to the natural philosophers to argue about" (KGW II 151–152).

[44]Kepler (2000, V 4, 223): "For as the picture is, so is the vision." Kepler reached this formula after having stated that the function of the crystalline is not to enlarge the painting on the retina, but to make it clearer. The object painted on the retina should not "occupy a greater quantity on the retina than is correct" (KGW II 186).

[45]For the genesis of the distinction between these two types of images, see Dupré (2008, 2012). In the last article, Dupré shows that Kepler appropriated some innovations found in 16th century treatises of mathematical optics but aimed to give the physical causes of optical phenomena and acted as a natural philosopher in optics.

[46]Kepler (2000, III 2, def. I, 77) (KGW II 64).

Whereas colors were seen as they were in the object, spatial properties like size and shape, but also distance as we will see shortly, could be altered in the *imago*.[47] The opposition between *imago* and *pictura* was clearly expressed by Kepler:

> Since hitherto an Image has been a Being of the reason, now let the figures of objects that really exist on paper or upon another surface be called pictures.[48]

By opposition to the *pictura* which had an almost material existence (as it is the case with the retinal picture), but was only distinctly visible on a screen on which it was projected, the *imago* or *imaginatio* was, for Kepler, an entity visible per se, perceived by the eye, but which did not have the physical reality of the *pictura*.[49] It was a mix of physicality and subjective intentionality. To this kind of optical entity belong the images reflected in mirrors or seen by refraction. Kepler's approach to the status of reflected and refracted images is therefore in the line of ancient and perspectivist optics.[50] Now Kepler addressed the problem of the perception of location and distance through vision precisely when he dealt with these *imagines* in Chap. 3 of his *Paralipomena*. Kepler sought to determine where the perceived image was located, that is to say where vision located the object seen by a reflected or refracted image.[51] For that purpose, he appealed to a kind of geometry of the triangle and the process involved a psychological approach to vision:

> Thirdly, since to each animal a pair of eyes is given by nature, with a certain distance between them, by this support the sense of vision is most rightly used to judge [*iudicandas*] the distances of Visibles […]. For here it is simply the geometry of the triangle […].
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> *For, given two angles of a triangle, with the side between them*, the remaining sides are given. In vision, the *sensus communis* grasps [*tenet*] the distance of its eyes through becoming accustomed to it, while it takes note of the angles at that distance from the perception of the turning of the eyes towards each other.[52]

While this geometry of the triangle did not essentially rely on a mental calculation or on determinate psychological faculties, it presupposed that the percipient had knowledge of the distance separating his eyes and the sensation produced by the rotation of his eyes orienting towards the object seen. This enabled the

[47]As I shall show, this is precisely the contrary in Descartes' optics where colors, as we perceive them, are not part of reality, whereas the properties of extension have a stabilized ontological status.

[48]Kepler (2000, V 3, def., 210) (KGW II 174).

[49]Simon (1979, 465): "l'image n'est rien d'autre que ce que l'on *voit*, elle n'a pas par elle-même une existence indépendante de *chose*." ("the image is nothing else than what is *seen*, it does not have by itself the independent existence of a *thing*.").

[50]Simon (1981, 306): "ce non-être existe dans notre imagination, mais projeté hors de nous, ce n'est qu'un fantôme. Voir une image, c'est voir la chose, mais là où elle n'est pas." ("this non-being exists in our imagination; but given that it is projected without us, it is only a phantom. Seeing an image is seeing the thing, but where it is not.") This statement is about Ptolemy's optics but can apply as well to the perspectivist tradition.

[51]On that aspect of Kepler's *Paralipomena*, see Simon (1979, 464–477).

[52]Kepler (2000, III 2, prop. VIII, 79) (KGW II 66).

observer to evaluate the two angles and the length of the intermediate side of a triangle formed by his two eyes and a point on the image of the object. By what resembles a calculation but which is not really one because it is more intuitive,[53] one could therefore determine the length of the two other sides, that is to say the distance between the percipient and the object.

As we have seen, the perspectivists gave a mainly psychological explanation of depth perception, but resorted to the cathetus rule to determine the locus in which the reflected or refracted image of an object is seen. Yet as Kepler showed, this rule was not valid for non-plane mirrors. Kepler's triangulation was therefore meant to replace the cathetus rule. He criticized the cathetus rule because it relied on a finalist principle.[54] Indeed, according to Witelo, "the object must appear on the perpendicular since we know…that this represents the shortest distance from either the surface of the mirror from which reflection is produced to the eye or the surface that forms its continuation. And along this perpendicular the object of sight maintains a uniform situation with respect to the mirror, and consequently the form of the object takes the designation of 'image' as we said before."[55] Kepler, on the other hand, considered that what was to be taken into account was the shape of the mirror at the point of reflection and the position of the two eyes in relation to this point. Since the eye has no possibility to determine where the ray comes from beyond the point of reflection, the sense of vision *imagines* that the object seen is located in the continuation of the reflected ray.[56] Kepler thus considered that the image of a point was located at the intersection of the reflected or refracted rays that would eventually reach the eyes. It is particularly noteworthy that Kepler then reintroduced the vocabulary of the visual ray: "And the genuine place of the image is that point in which the visual rays from the two eyes meet, extended through their respective points of refraction or reflection…Therefore,

[53]The mode according to which distance is apprehended through vision is expressed by the Latin verb *tenere*. This verb refers more to an act of grasping than to a step by step calculation. This is even more obvious in the passage on the triangle based on one eye where Kepler opposed grasping (*tenere*) and numbering (*numerare*), rejecting the latter: see below.

[54]Kepler (2000, III 1, 75): "[Alhazen] nevertheless seems to be implying that this location of the image on the perpendicular was long ago thus established by God the Creator because it would be best so, and no more fitting place could be given to the image…And in fact all these affects are consequences of vision by material necessity, where considerations of purpose or beauty have no place." (KGW II 63) On Kepler's criticism of the cathetus rule, see Chen-Morris and Unguru (2001).

[55]Witelo (1983, V 37, 122).

[56]Kepler (2000, III 2, prop. XVII, 85): "First, the sense of vision [*visus*] errs in direction…: it imagines for itself [*imaginatur sibi*] an object in the same direction whence the refracted or reflected ray approached. Next, the sense of vision also errs in the angle. For it imagines for itself that the inclination by which the refracted or reflected rays proceed all the way to the centers of the two eyes, is also the same as the inclination or angle by which proceed those rays which approach from the radiating point to the points of the reflections or refractions, corresponding to the eye…" (KGW II 72).

since the place of the image is in the meeting of the visual rays [*radii visorii*],…it will be in the meeting of the surfaces of refraction or reflection of the two eyes."[57] The visual ray is defined as "the luminous line drawn out by the imagination from the eye continuously through the point of reflection or refraction."[58] The light ray is thus extended by imagination beyond the point of reflection as if it were a visual ray. The capacity to locate images in the visual space therefore depends on the ability the imagination has to project, in a representative way, imaginary lines from the eyes to the point of reflection or refraction and beyond it in the visual space opened by the mirror or the refractive lens. Moreover, Kepler mentioned the *sensus communis* in the procedure of triangulation, a notion coming from Scholastic psychology and which seems here to refer to a function of the mind not assigned to a specific part of the body. One has to notice that in the analysis of depth perception for objects indirectly seen through *imagines*, Kepler did not appeal to imagination as a specific faculty, integrated in an ordered, multileveled psychological system in which it would be attributed a particular cognitive function (as this was the case in Scholastic and in the perspectivist traditions). For Kepler, imagination was on one hand the image seen, that is to say an object of vision, and on the other hand the activity of the sense of sight (and not so much of a proper faculty that would be called "imagination") when it illusorily locates the object of the image produced by reflection or refraction.

Now, in propositions 9–14 of Chap. 3, Kepler presented another way to evaluate distance that implied only one eye and that relied on a psychological function which can be considered as assigned to a part of the body. This second way of evaluating distance relied on a type of psychology immanent to the eye. Kepler considered that, depending on the distance of the image, a more or less dense quantity of light (that is to say more or less dispersed by a more or less wide beam)[59] entered the eye and was projected on the back of the eye. This projection would be more or less wide and therefore more or less dense depending on the quantity of light reaching the eye. The further away the object seen, the narrower the *pictura* projected on the retina.[60] The eye could then sense the density of light,[61] the size of this projection, as well as the geometrical configuration linking the extremities of this projection to the edges of the pupil. By being extended, these lines intersect in a point that indicates the distance at which the point on the object seen is located. Even though Kepler denied that the retina could "see" the retinal *pictura*, he considered that the eye could sense the density of light, the

[57]Kepler (2000, III 2, prop. XVII, 85) (KGW II 72).

[58]Kepler (2000, III 2, prop. XVII, 85) (KGW II 72).

[59]Kepler (2000, III 2, prop. XIII, 82): "For while light passes through this depth, it is spread out in a certain proportion…" (KGW II 69).

[60]Kepler (2000, III 2, prop. XIV, 83) (KGW II 70).

[61]Kepler (2000, III 2, prop. XI, 81): "it is fitting that there be in the eye the power of measuring either the density or rarity of both the air and the light." (KGW II 68); Kepler (2000, III 2, prop. XII, 82): "therefore, *the eye too will perceive the density of light*" (KGW II 69).

width of the retinal projection,[62] the distance between the retina and the pupil, and the pupil's diameter.[63] He endowed the eye as a whole and in a vague way[64] with a sensorial power of which he had precisely deprived the retina as a clearly identified physiological element. In other words, with the triangle based on one eye and not on binocular vision, as well as on the movement of the eyes turning towards each other to aim at what is seen, Kepler reintroduced a sensorial faculty in a diffuse way in the body, or at least in that part of the body which is the eye,[65] in a way similar to that of the perspectivists. It is important to note, especially in view of a clear difference with Descartes, that Kepler considered that the evaluation of the lengths of the sides of the triangle giving the distance of the point on the object in space was not subject to a calculus, but to an observation of the eye.[66] Kepler therefore assigned to multiple more or less decentralized instances the sensitive power which could be disseminated through various parts of the body.[67] This means that his psychology of vision was not a thoroughly coherent and centralized one.[68] This would also amount to an important difference with Descartes for whom the soul, and not the body, was the sensorial agent.[69]

[62]Kepler (2000, III 2, prop. XIII, 82): "we attributed to it a perception of the quantity of the surface touched by light." (KGW II 69) In proposition 61 of his *Dioptrice*, Kepler defined vision as "the sensation of the affected retina which is full of visual spirit; or seeing is sensing the retina being affected insofar as it is affected." (KGW IV 372: "Visio est sensio affectae retiformis spiritu visivo plenae: sive, Videre, est sentire affectam retiformem, quatenus affecta.").

[63]See Kepler (2000, III 2, prop. XIV, 83) (KGW II 69).

[64]Kepler's indecision is manifest in proposition XII. Kepler (2000, III 2, 82), my emphasis: "why therefore should *the eye or sense of vision* in general not also receive something contrary from this density of light, likewise in relation to its own density?" (KGW II 69).

[65]Kepler (2000, III 2, prop. X, 80–81): "For this faculty of setting up the measuring triangle is common to the two eyes together and to each eye separately" (KGW II 68).

[66]Kepler (2000, III 2, prop. XIV, 83): "it will consequently observe $\alpha\eta$ and $\alpha\vartheta$, not, indeed, by numbering but by comparing the distances of the object through this habit, as it were, with the powers of its body, and the extension of hands and of paces" (KGW II 70).

[67]Simon (1979, 562): "[Kepler] multiplie les facultés psychiques ou leurs équivalents à l'intérieur du globe oculaire." ("[Kepler] multiplies the psychological faculties or their equivalents inside the eyeball.").

[68]Simon (1979, 563): "son panpsychisme lui permet, dans chacun des cas, de multiplier à sa convenance les zones et les formes de sensibilité, il le conduit à atomiser le sujet de l'acte de vision: au fil de la plume, ce qui 'sent', ce qui 'connaît', ce qui 'mesure', ce qui 'perçoit', ce qui 'compare', ce qui 'a l'habitude', ce qui 'estime', ce qui 'juge', est indifféremment une membrane de l'œil, l'œil lui-même, le sens commun ou la faculté visuelle, quand ce n'est pas tout simplement (et c'est le plus fréquent) *visus*, la vue." ("his panpsychism allows him, in each and every case, to multiply the areas and forms of sensibility as he wishes; it leads him to atomize the subject of the act of vision: in the course of writing, what 'senses,' what 'knows,' what 'measures,' what 'perceives,' what 'compares,' what 'is accustomed to,' what 'estimates,' what 'judges' is indifferently an eye membrane, the eye itself, the common sense or the visual faculty, when it is not very simply (and this is the most frequent case) *visus*, eyesight.").

[69]See Descartes (1984–1991, I, 164) (AT VI 109: "c'est l'âme qui sent, et non le corps").

Even if the eye was not able to perform a real calculus, Kepler conceived it, as is manifest in his analysis of images, as endowed with a power to recognize some mathematical relations in space. This means that, in Kepler's theory, there is no actual explicit inferential procedure. In that case, how is it possible for the eye or the sense of vision to "sense" or imagine these spatial mathematical relations? One possible answer lies in Kepler's theory of knowledge insofar as it aimed to renew the relations between sense perception and mathematics.[70] This is more explicit in Kepler's theory of harmonies: the human mind does not abstract mathematical ratios from the sensible but is itself endowed with mathematical knowledge. In his *Harmonice mundi*, Kepler thus wrote: "Proportions are entities of Reason, perceptible by reason alone, not by sense, and to distinguish proportions, as form, from that which is proportioned, as matter, is the work of the mind."[71] The key to such mechanisms resides in the fact that "the soul has knowledge of mathematics by instinct."[72]

> Indeed to the human mind and to other minds quantity is known by instinct, even if for this purpose it is deprived of all sensation. Of itself it understands a straight line, of itself an equal distance from a given point, of itself it forms for itself from these an image of a circle. If so, it can much more readily find the construction by means of that, and so perform the function of the eye in seeing the diagram (if there is nevertheless a need for one).[73]

This mathematical instinct seems to exclude any properly discursive reasoning (and it is common to human beings and to animals here designated as "other minds"). This is certainly a reason why it involves an imagining activity rather than the intellect.[74] The soul can spontaneously construct the image of a circle.[75] It can also, by imagination, take on the function of an eye contemplating a diagram. Moreover, the eye can only see the geometrical diagram because it was conceived in conformity with the soul. The eye sees because it is functionally patterned on the soul, and in particular on the soul's capacity for inner visualization. In other words, the eye was patterned on imagination. To summarize, visual perception works in a way that depends on the mind, which itself is imprinted with mathematics. The eye is meant to be used by the mind in order to recognize mathematical entities in nature:

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[70]On this topic in general, see Chen-Morris (2001). This article gives a thorough analysis of this question in relation to its Aristotelian background. I rely on it in order to link this topic to the evaluation of the distance of images in Kepler's optics. See also Escobar (2008).

[71]Kepler (1997, III. I, 150) (KGW VI 107).

[72]Kepler (1997, IV. I, 303) (KGW VI 223: "Anima habet scientiam Mathematum ex instinctu").

[73]Kepler (1997, IV. I, 303–304) (KGW VI 223).

[74]We shall see that the involvement of imagination in the Cartesian natural geometry obeys a quite different logic (since it is reserved for human beings).

[75]In the *Harmonice mundi*, Kepler indeed claims that the circle is like the form of the created soul: see KGW VI 277 ("ut forma quaedam ipsius Animae"); Kepler (1997, 373).

> Certainly the mind itself, if it never had the use of an eye at all, would demand an eye for itself for the comprehension of things which are placed outside it, and would lay down laws for its structure which were drawn from itself…For the recognition of quantities, which is innate in the mind, dictates what the nature of the eye must be; and therefore, the eye has been made as it is because the mind is as it is, and not the other way round.[76]

By means of sensation, the soul recognizes in the world its own nature, what properly belongs to it, that is to say mathematical entities produced by the imagination.[77] The human mind can thus see itself reflected in sensible perceptions insofar as it informs the sensorial faculties. One understands then the possibility the mind has to project visual intentional rays in visual space so that it can recognize there the mathematical relations that will allow it to determine the location of an *imago*.

In the two ways of evaluating distance by sight, Chap. 3 of the *Paralipomena* brought into play a physiological and psychological dimension of vision, before Kepler dealt with the eye and the *modus visionis* in this chapter. Kepler thus rejected the cathetus rule but conserved an intentional dimension to the perception of the distance of refracted or reflected images that came from perspectivist optics. This dimension is mostly manifest in the mention of visual rays and in a judgment performed by the sense of vision. On the contrary, Kepler gave a mostly physical account of the *pictura* by searching to locate the source of divergent light rays that produce this kind of image.[78] *Imago* and *pictura* therefore gave rise to two distinguished optical explanations. The psychological account of the distance of reflected or refracted images was developed within a theoretical framework which obviously escaped from Keplerian optical reform based on the theory of retinal picture formation. If both images involved a geometrical analysis, the *pictura* referred to physical reality,[79] whereas the *imago* was no more than an illusion with respect to the real situation of the object in space. This disjunction shows that Kepler, from that point of view, remained the heir of a traditional optics which considered that reflected or refracted images showed objects where they were not,

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[76]Kepler (1997, IV. I, 304) (KGW VI 223).

[77]See Claessens (2011).

[78]For the posterity and the reinterpretation of these two dimensions of Keplerian optics into real and virtual images, see Shapiro (2008).

[79]The retinal picture can be analyzed in geometrical terms and is liable to geometrical definition through the very way it is formed from light rays being refracted in the eye. This mathematical dimension explains why it can faithfully reproduce the external world: see Malet (1990). J.V. Field even goes as far as to claim that Kepler's confidence in the fact that physical things behave like mathematical ones can account for his adoption of a reversed and inversed retinal picture as what provokes vision: see Field (1986). The mathematical dimension is also central to Kepler's analysis of reflected and refracted images, but it involves an intentional dimension that makes it less certain and more liable to errors.

that is to say fell under visual illusions.[80] The evaluation of the distance of these images in a kind of virtual, purely visual, space could therefore not be confused with the evaluation of the distance of objects in their true place and in their true spatial properties.[81] This is indeed very telling that, when Kepler dealt with the evaluation of distance by direct vision in his *Dioptrice*, he mainly remained within the theoretical framework of traditional optics, taking into consideration the visual angle, as in Euclid, and the relation between the size attributed to the object and its unknown distance, but appealing to no operation of triangulation as Descartes would do.[82] Moreover, for Kepler, even the rules according to which the distance and location of reflected and refracted images could be determined were liable to errors.[83] In Kepler's optics, the psychological process of distance measuring by triangulation could in no way constitute the basis of an epistemological foundation of vision. In the final part of this paper, I want to show that Descartes precisely took up the Keplerian distance-measuring triangle, but applied it to another subject: not to the distance of images but to the distance of objects themselves as they are seen through direct vision. This brought about an important epistemological reversal in Descartes' theory of vision.

[80]For example, Pecham defined the images reflected in mirrors as follows: "What then is an image [*ydolum*]? I say that it is merely the appearance of an object outside its place [*apparitio rei extra locum suum*]…it is the object that is really seen in a mirror, although it is misapprehended in position [*in situ erratur*] and sometimes in number…"(Pecham 1970, II 19, 171).

[81]Simon (1979, 584–585): "Tant que, derrière la distinction entre *pictura* et *imago*, se profile le réalisme intellectualiste de la pensée médiévale, l'image étant due à un jugement reste une entité psychique, et ne peut, comme effet d'un rayonnement, matérialiser fictivement une origine extérieure: sa 'virtualité' est dans la tête de l'observateur, non dans l'espace qui lui fait face." ("As long as the intellectual realism of medieval thought looms behind the distinction between *pictura* and *imago*, the image arising from a judgment remains a psychological entity and cannot, as the effect of light radiation, materialize its outward origin in a fictitious way: its 'virtuality' resides in the observer's head, not in the space facing him.").

[82]See *Dioptrice*, propositions 67–68, KGW IV 376–377.

[83]Hamou (1999, 215): "[Kepler chercha à] lui substituer une règle d'assignation de la distance et de la 'quantité' (grandeur) de l'image qui met en jeu le facteur intentionnel…(l'œil est attiré spontanément vers la lumière et juge de la direction des objets en fonction de celle des rayons qui lui parviennent) et un jugement trigonométrique implicite…Cela étant, la règle de localisation des images proposée par Kepler…ne permet pas de comprendre l'image vue par réfraction comme pourvue d'un lieu géométrique réel. Non seulement la localisation est l'effet d'un jugement où intervient une connaissance naturalisée, une habitude, et donc est éminemment susceptible d'erreur, mais, comme le montre l'étude 'phénoménologique' des *Paralipomènes*, il y a plusieurs circonstances qui font que la règle parfois ne s'applique pas." ("[Kepler sought to] replace [the cathetus rule] with a rule by which to assign the distance and 'quantity' (magnitude) of the image involving an intentional factor…(the eye is spontaneously attracted to light and judges of the direction of objects according to that of the rays that reach it) and an implicit trigonometric judgment…That being so, the rule of localization of images proposed by Kepler…does not allow one to understand how the image seen through refraction could be located in a real geometrical locus. Not only is the localization an effect of a judgment involving some naturalized knowledge, some habit, and is therefore eminently liable to error, but, as shown by the 'phenomenological' inquiry of the *Paralipomena*, in several circumstances the rule sometimes does not apply.").

5.4 The Perception of Distance Through a Natural Geometry in Descartes' Optics

Descartes, who was usually very reluctant to acknowledge his sources, did not hesitate to declare to Mersenne that Kepler was his "first master in optics."[84] Descartes' *Dioptrique* indeed took up Kepler's account of the retinal image and attempted to determine the conditions that enabled it to be as accurate and complete as possible. But, what really mattered for Descartes was not so much the retinal picture itself as the judgment produced in sense perception out of which we determine, from what we see, that the external object located in space is endowed with a given property (among which that it is situated at such a distance and possesses such a shape).[85] But the optical analysis of the retinal picture did not suffice to account for all the dimensions of the bodies perceived. In particular, the problem of the perception of spatial three-dimensionality remained to be solved. For that purpose, Descartes appealed to a procedure which did not belong solely to physical optics. Within the realm of the latter, one could at best have explained through which procedures we could perceive a picture of colored spots in two dimensions that would be similar to the retinal picture.[86] But how could one account for the sense of spatial depth which is always associated with our visual perception? How could one explain that the light rays that reach the eyes also make us see the shape of the object in three dimensions?

Descartes added to his mechanical optics (according to which a picture is imprinted, after several refractions, on the retina and movements are transmitted to the brain and the pineal gland) a *natural geometry* required to make us access the visual perception of shape, distance and depth. In Discourse VI of his *Dioptrique*, he gave a specific treatment of the situation, distance, size and shape of the bodies as they are perceived through vision. He used the expression "natural geometry" only to describe the evaluation by sight of the distance at which bodies are situated:

> In the second place, we know distance by the relations of the eye to one another. Our blind man holding the two sticks AE and CE (whose length I assume he does not know) and knowing only the distance between his two hands A and C and the size of the angles ACE and CAE, can tell from this knowledge, as if by a natural geometry, where the point E is. And similarly, when our two eyes A and B are turned towards point X, the length of

[84]Descartes to Mersenne, 31 March 1638, AT II 86.

[85]On this topic, see Descartes' *Replies to the Sixth Objections* (AT VII 436–439; AT IX-1 236–238) and my comment in Bellis (2010, 376–401).

[86]I am not here saying that Descartes conceived of the retinal picture as giving rise to a kind of rough sensation of colored spots in two dimensions that would be elaborated upon by a psychological process. Actually, for Descartes, our sensations always involved, right from the start, the perception of three-dimensional space. In his *Reply to the Sixth Objections*, Descartes seemed to distinguish between these two levels of sensation, the more complex one involving a judgment. But, as I have shown elsewhere, the distinction is only the result of an analytical explanation that was not intended to give a genetic account of sense perception. See Bellis (2010, 383–387).

> the line AB and the size of the two angles XAB and XBA enable us to know where the point X is…And this is done by a mental act which, though only a very simple act of the imagination, involves a kind of reasoning quite similar to that used by surveyors when they measure inaccessible places by means of two different vantage points.[87]

This natural geometry relies on a distance-measuring triangle very similar to that of Kepler's *Paralipomena*[88] and consists in a kind of calculation. But Descartes displaced the field of application of this psychological procedure from reflected and refracted images to bodies seen by direct vision. As a consequence, natural geometry does not only refer to a question of psychology, but also to a question of epistemology, since what is at stake is to know whether and under which conditions vision can allow us to situate objects in space according to their true distance, location, size, and shape.[89] From that epistemological point of view, the very idea of a geometry is revealing since natural geometry ought to have its principles and rules, just like mathematical geometry. Both were instituted by God, either through the institution of Nature (as the term "natural" qualifying this geometry suggests), or through the free creation of eternal truths.[90] In the case of natural geometry, its principles correlate the distance between our eyes—a distance which is inscribed in our body—and the movement of the eyes required to see an object situated in a determinate part of the visual space with the distance at which this object is situated. There is almost a kind of mathematical relation between bodily data and objective properties of the material extended reality. The idea of a natural geometry also suggests that this psychological process is based, as geometry in general is for Descartes, on innate ideas. These are the bases for natural geometry, enabling us to recognize geometrical figures in space.[91]

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[87]Descartes (1984–1991, I, 170; AT VI 137–138).

[88]See de Buzon (1991, 98–99 n. 21)

[89]The determination of the shape and size of bodies indeed ultimately relies on that of distance and location: see below. My epistemological interpretation of the natural geometry therefore differs from Celia Wolf-Devine's interpretation: see Wolf-Devine (1993, 76–77). It also opposes Malet's skeptical reading of the power of the senses as presented in the *Dioptrique*: see Malet (2001). Instead of considering that "Descartes held our eyes to be hardly reliable at measuring distances" (Malet 2001, 129) because he stated that the various procedures at hand were reliable only below certain distances, I think that Descartes precisely gave the restricted conditions under which it is possible to see the spatial properties of things as they are. This restriction is not a skeptical perspective, but rather the establishing of the conditions under which eyesight faithfully gives us access to the spatial properties of extension. That optics in the early modern period involved important epistemological stakes can be seen for example in the way optics was defined as "the art of seeing well," which means that its aim was to judge of the truth or of the fallacy of what is seen. This statement is found in Risner. Risner (1606, 3): "Optica est ars bene videndi. Optica suo fine definitur, qui est bene videre, id est, de veritate & fallacia visibilium accurate and exquisite judicare." ("Optics is the art of seeing well. Optics is defined by its aim which is seeing well, that is to say judging precisely and in a thought-out way the truth and falseness of visible things.").

[90]See Descartes to Mersenne, 15 April 1630, 6 May 1630, 27 May 1630, AT I 145–146, 149–150, 151–153.

[91]See Descartes' *Reply* to Gassendi's *Fifth Objections*, AT VII 381–382.

However, does the percipient actually appeal to an effective geometry or is this only an analogy? In the *Dioptrique*, just as in *L'Homme*,[92] the expression was used about the blind man holding two sticks and came into play within a comparison ("*as if* by a natural geometry"). At first sight, it is an analogy within an analogy, or a second-order analogy. As a consequence, what importance should we give to such a notion? Is a true geometry really involved in sensation? Is natural geometry really a geometry? I shall argue that this is really the case, but precisely insofar as this geometry falls under the institution of nature. By this institution, our physiological mechanism is constituted in such a way that it makes the soul sense the essential properties of the bodies seen, on the basis of a correlation between on one hand a modification of the pressure exerted on the optical nerve, the movements of the small strings enclosed in the nerves, the position of our eyes and of our body in space, etc., and on the other hand what our soul senses in terms of the distance of the object perceived. For Descartes, God has imprinted in our body and in our soul a certain type of reaction according to the data linked to the position and shape of our body (or of some of its parts) and the surrounding bodies.

But a "reasoning" is also involved. Through natural geometry, Descartes wanted to conceive a psychological procedure by which the percipient could sense the spatial three-dimensionality, and not only rough two-dimensional visual data made of colored and luminous spots. The introduction of a natural geometry signifies that, for Descartes, sensing spatial depth did not fall under raw visual data. It also means that it was not obtained through an inference that would associate, on the basis of repeated experiences, the vision of two-dimensional luminous and colored spots, or the sensation of ocular movement, with the evaluation of distance through, for example, the displacement in space of a subject who would, afterwards, touch objects where they actually stood. Vision in perspective is not the result of an apprenticeship correlating plane vision and touch, but it is produced thanks to bodily conditions and to a trigonometric reasoning, thus to geometrical notions.[93]

In the *Dioptrique*, natural geometry was one of three ways to determine the distance at which an object is seen.[94] However, the whole of the psycho-physiological processes at stake in vision eventually relied on a natural geometry. Indeed, according to Descartes, the determination of the shape and size of bodies

[92]See AT XI 160.

[93]Simmons (2003, 398): "These judgments effectively recover the three-dimensional properties of objects and explain perceptual constancy: they explain why things look to have constant shapes and sizes despite the fact that the portion of the visual field they fill is constantly changing as we move through the environment".

[94]The two others respectively rely on a change in the shape of the eye (accommodation) and on the more or less important degree of distinction and of luminosity of the sensible representation: see AT VI 137–140.

ultimately relied on the determination of the situation and distance.[95] Regarding the evaluation of situation, it might first appear as a purely bodily, thus mechanical, process: "Our knowledge of it does not depend on any image, nor on any action coming from the object, but solely on the position of the tiny parts of the brain where the nerves originate."[96] But the following part of the text indicates that this operation goes beyond mere mechanism, insofar as it implies a geometrical understanding of visual space inherited from the Euclidean tradition (including what the visual ray aims at):

> For this position changes ever so slightly each time there is a change in the position of the limbs in which the nerves are embedded. Thus it is ordained by nature to enable the soul not only to know the place occupied by each part of the body it animates relative to all the others, but also to *shift attention* from these places to any of those lying on the straight lines which we can *imagine* to be drawn from the extremity of each part and extended to infinity.[97]

This geometrical structure of visual space by rectilinear projection is the result of a quasi-intentional activity of the mind by which an object is aimed for within a natural geometry.[98] On the one hand, light acts upon the eye according to physical rays which are nothing else than the communication, in a continuum of subtle matter, of the pressure coming from the movements of particles of the first element in the Sun to the observer, these rays being liable to reflection or refraction through their encounter with visible objects. These rays correspond to the lines along which a real physical action is produced. On the other hand, the imagination can trace in the surrounding extension some imaginary, mathematical and not physical, lines by which thought can situate objects in space, in relation to my body and to other bodies. The analysis of vision that Descartes proposed thus effectuates an organization of material reality according to geometrical lines which correspond to a physical reality as well as to a psycho-spatial reality. Admittedly, these lines are not perfectly identical, since the physical lines ultimately refer to invisible corpuscular phenomena by which light rays are propagated, and the geometrical lines correspond to the sensible grasp of the external spatial reality. Moreover, even if one considers light propagation in straight lines at the macroscopic level, imagination goes past the geometrical line followed by the reflected ray of light and extends the imaginary line beyond the object. This enables the percipient to locate

[95]Descartes (1984–1991, I, 172): "Concerning the manner in which we see the size and shape of objects, I need not say anything in particular since it is wholly included in the way we see the distance and the position of their parts" (AT VI 140).

[96]Descartes (1984–1991, I, 169); AT VI 134.

[97]Descartes (1984–1991, I, 169, my emphasis); AT VI 134–135.

[98]Hyman (1986, 160): "…the situation of an object or a part of one is naturally identified by means of a deictic gesture, by pointing, and not by means of a description, and so the visual perception of situation is perfectly attuned to the walking-stick analogy".

Fig. 5.1 The blind man and the perception of distance (AT VI 135), in *Renati Descartes Specimina Philosophiae*, Amsterdam: Blaeu, 1685, p. 87 (reproduced with kind permission of the University Library of the Radboud University Nijmegen)

the object in extended space in general, and not only according to the punctual spatial relation of my body to this object (Fig. 5.1):

> In the same way, when the blind man, of whom we have already spoken so much, turns his hand A towards E, or again his hand C towards E, the nerves embedded in that hand cause a certain change in his brain, and through this change his soul can know not only the place A or C but also all the other places located on the straight line AE or CE; in this way his soul can turn its attention to the objects B and D, and determine the places they occupy without in any way knowing or thinking of those which his hands occupy.[99]

Natural geometry thus allows us to access a whole perceptual space with multiple relations. The lines in the perceptual space are drawn by the imagination which also locates objects within this space. By opposition to Kepler's triangulation in Chap. 3 of his *Paralipomena*, here the imagination allows the mind to project itself in a non-illusory way in real extended space (and not only in an imaginary or virtual space opened by the mirror). Admittedly, the location and distance of objects is evaluated in relation to my own body.[100] But Descartes here suggests that imagi-

[99]Descartes (1984–1991, I, 169); AT VI 135.

[100]Simmons (2003, 400): "What Descartes and Malebranche are latching onto here is the fact that in sensory experience the perceiver's body effectively fixes the origin and the axial symmetries of the space within which objects appear to be located: my body is always located as *here* and objects appear as situated *around me*, at some distance and direction from here." But Simmons does not take into account the capacity of projection of the imagination beyond the punctual spatial relation of my body to the object aimed at by eyesight. Moreover, the fact that the operation by which I locate objects in space is always made from a specific point of reference does not constitute, for all that, a hindrance in gaining access to the objective space through eyesight. This objectivity is precisely guaranteed by natural geometry, in the same way as a two-dimensional painting seen from a given point of view constitutes a basis from which I can reconstitute a three-dimensional landscape whose dimensions do not depend only on my particular situation in relation to the painting.

nation is endowed with a capacity to overstep the mere relation of my body to the object that I see by projecting itself in extension beyond a given object.

As Descartes intimates as a possibility in the text of the *Fifth Meditation*, I can indeed project the essence of matter (that is to say extension or geometrical space) into my sensations by means of imagination. Geometrical extension is present in my mind as an innate and imaginable idea. Therefore, I can reconstruct the material objects as they are perceived in vision thanks to this faculty of projection which lies in the imagination. Imagination is the mode of thinking by which I represent to myself the "'continuous' quantity as the philosophers commonly call it," that is to say "the extension of the quantity (or rather of the thing which is quantified) in length, breadth and depth."[101] If imagination is required in natural geometry, this is because it is required to represent spatial depth in general, but above all the particular aspects of spatial depth:

> I also enumerate various parts of the thing, and to these parts I assign various sizes, shapes, positions and local motions; and to the motions I assign various durations.
>
> Not only are all these things very well known and transparent to me when regarded in this general way, but in addition there are countless particular features regarding shape, number, motion and so on, which I perceive when *I give them my attention* [*attendendo percipio*]. And the truth of these matters is so open and *so much in harmony with my nature* [*naturae meae consentanea*], that on first discovering them it seems that I am not so much learning something new as remembering what I knew before; or it seems like noticing for the first time things which were long present within me although I had never *turned my mental gaze on them* [*in illa obtutum mentis convertissem*] before.[102]

Instead of giving an almost Platonist interpretation of this passage, I propose to interpret it as echoing the Cartesian theory of natural geometry. Admittedly, mathematical ideas are innate in our mind. But the application of the mind's attention can here be interpreted not only according to pure intellection, but also as the specific effort of the mind that is proper to imagination. Moreover, if the particular truths I discover about numbers, shapes, and movements are "in harmony with my nature," this is because these mathematical truths are *mentibus nostris ingenitae*.[103] But these "particular features" are also imprinted in our body—even if this body's existence has not yet been established at this moment of the *Meditations*. These features are perceived by vision through a process of projection of the imagination in spatial extension thanks to which we measure and locate the various parts of the surrounding bodies. In this process, no iconic element is presupposed as a prerequisite to effective vision (as was the case with Scholastic *species*). On the contrary, in vision the mind itself depicts the visual picture. And imagination comes into play precisely at this level. In other words, the imagination's activity replaces the images-species' objectivity. Apart from the soul, there is only body and body cannot project itself into space, except physically by motion. The Cartesian conception of vision is therefore not deprived of intentionality but the latter is performed by

[101] Descartes (1984–1991, II, 44); AT VII 63.

[102] Descartes (1984–1991, II, 44 my emphasis on the English translation); AT VII 63–64.

[103] Descartes to Mersenne, 15 April 1630, AT I 145.

imagination which is the mode *par excellence* of spatial representation. Mathematical imagination possesses a dimension of intentionality insofar as it allows the mind, in visual sensation, to hone in on corporeal extension in its particular features, but also in its more general features, since Descartes considers that it is possible to imagine lines "extended to infinity." Thanks to the role played by imagination in vision, vision is not a truncated representation of spatial extension, but encompasses it as an infinite potential space.

Thus, there exists in the *Dioptrique*, apart from the mechanical processes linked to the tendency to motion of the subtle matter exerted on the eye, another aspect of vision, which is not purely mechanical but is nevertheless geometrical. Whereas Kepler subordinated the psychological dimension of vision to the identification of the cause of our perceptual errors concerning reflected or refracted images, Descartes fully endorsed that, in non-deceptive direct vision, a psychological dimension, in addition to the purely physical dimension, comes into play. This does not mean that Descartes just came back to the theories of the visual ray or to the perspectivist or Scholastic theories which analyzed vision on the basis of species, a mix of physical and psychological reality that transported the object's resemblance and made it visible as it is.[104] There is no confusion any more between the study of the propagation of light and the theory of vision. However, the visible and, in particular, the spatial depth, became subjected to a mental reconstruction that involved the psychological activity of the subject. This psychological dimension does not appear as an obvious component that naturally accompanies the transmission of light and species, but as a required dimension of vision, *additional* to the study of light propagation which, because it has become in the meantime autonomous, cannot account anymore for spatial depth. Because of the real distinction between the soul and the body, Descartes claimed: "it is the soul that has sensory perceptions, and not the body."[105] There is now one psychological instance involved in the process of vision, the soul. This marks a noticeable difference with Kepler who multiplied the sentient instances.[106] This is the reason why Descartes, if he took up Kepler's triangulation, did not take up the distance-measuring triangle based on only one eye, but instead displaced the eye at two different points of observation (which amounts to reconstituting binocular vision in the imagination).[107] The Keplerian explanation supposed that the eye itself evaluated the width of the luminous projection on the retina—which Descartes would

[104]See Hamou (2002, 39–44).

[105]Descartes (1984–1991, I, 164) (AT VI 109: "c'est l'âme qui sent, et non le corps").

[106]Simon (1979, 570): "La sensibilité indistincte et instruite dont on parsemait quasi par inadvertance les organes des sens jusqu'à leurs plus infimes parties, a définitivement disparu." ("The indistinct and informed sensibility that was diffused, almost inadvertently, throughout the sense organs unto their most minute parts has definitively vanished.").

[107]See AT VI 138.

exclude[108]—and perceived lengths. For Descartes, that would then amount to assuming that the eye, in a way, thinks, which was impossible for him.

Descartes' natural geometry represents a new step in the conceptualization of the evaluation of distance by eyesight. By distinction with Alhazen, the reasoning implied in the natural geometry of vision is clearly embedded in bodily capacities (since it is based on the distance between the two eyes and the movement of rotation of the eyes). This will open the way to Malebranche's "natural judgment."[109] But at the same time, it relies on a centralized psychological instance that effectuates all the operations of sensation, imagination, and reasoning. This sets Descartes apart from the perspectivist and the Keplerian psychology of vision, even if they all considered that a kind of judgment was involved in the process of perceiving distances.

5.5 Conclusion

Descartes therefore applied to vision (understood as the mental representation of an object located in three-dimensional space) a type of explanation that Kepler applied to images that had mainly a mental reality in virtual space. Descartes took into full consideration the psychological dimension of direct vision and reintegrated it into a theory which aimed to account for, not illusory images, but images corresponding to real extended space. The procedure that guarantees the correspondence between vision and reality belongs to what, for Kepler, was illusorily projected by the mind in visual space. Descartes relied on Kepler to produce an epistemological reversal as to the reliability of vision. For Descartes, an object is seen in the way an *imago* was seen for Kepler.[110] The displacement operated by Descartes not only implies a new psychology of vision, but also an epistemological foundation of vision that attempts to bridge the gap between the thinking subject and the extended world, or between visual space and extended space.

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[108]Descartes (1984–1991, I, 167): "Now, when this picture thus passes to the inside of our head, it still bears some resemblance to the objects from which it proceeds. As I have amply shown already, however, we must not think that it is by means of this resemblance that the picture causes our sensory perception of these objects—as if there were yet other eyes within our brain with which we could perceive it" (AT VI 130).

[109]See Hamou (2002, 120–122).

[110]Opticians after Descartes will rather accomplish an inverse move in considering that an image is perceived as an object that is to say at the place where the light bundle seems to diverge. See Shapiro (2008).

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