"A new agenda for the history of early 20th century (western) mathematics: re-examining those excluded from the dominant narrative of mathematical modernity"

First, I have to thank Catherine for her insistent invitation. But I have to note an intimidating fact: I am the only one who will talk about western mathematics in this session. Does that mean that western mathematics are not relevant to the theme of our workshop? I don't think so, quite the contrary as I'll try to show it.

I wanted to begin with noticing two recent events in the field of history of "modern" mathematics, that is late 19th and 20th century mathematics.

A first one is a special issue of *Science in context* in 2004 devoted to the history of modern mathematics, introduced by an article by L. Corry entitled: "The history of modern mathematics. Writing and rewriting". The second one is an international workshop "Between Modernism and Applications. Comparative Studies in the History of early 20th Century Mathematics", which took place in 2009 in Germany.

Both of them, which focused on the need of rewriting history and taking into account fields as applied mathematics, give evidence of new agendas for the historians of mathematics who work on the period 1870s-1930s.

I have to notice a third one, which takes place just now in Paris, a workshop "What Makes Modern Mathematics Different from Classical Mathematics ?" with philosophers and historians of mathematics, some of them I will mention and debate in this talk. A proof, if necessary, that mathematical modernity is today a debated question in historians' agenda.

But, what is this need of re-writing history? For what goals ? What issues? That's what I want to develop first.

I. The decades 1960-1980 : « the royal road to me »

The first narratives of early 20th century mathematics have been written in the 1960s by some mathematicians who had been students and have begun their mathematical life in the twenties and the thirties. Unsurprisingly, the history they wrote – mostly for mathematicians - was in the purest tradition of Whig history, laying out "the royal road to me", as described by I.G.G.

One name and one work symbolize this writing: Bourbaki and his *Eléments d'histoire des mathématiques* published in 1960.

Bourbaki is a pseudonym for a group of mathematicians, founded in France in the 1930s. This foundation was a response of French young academics to the mathematics developed and taught by the "patrons" of French mathematics. They thought that the patrons ignored most important innovations developed in other countries, especially in Germany. They undertook to write a new treatise built on these innovations which were improving several fields of mathematics, innovations and fields totally peripheral in French mathematics in these decades.

This refoundation of mathematics the Bourbaki had undertaken from the 30s, has an historical part, historical notes which were compiled and published in 1960 as a single volume. In the forward of their book, they explain how their historical notes deal foremost with the parts of mathematics they have already rebuilt.

They add a methodological precision I want to underline because of the success it had in history of mathematics : "The lecturer will find in these Notes nearly any biographical or anecdotic information about the mathematicians in question; above all, we tried to show as clearly as possible, for each theory, which have been the main ideas and how these ideas have developed and reacted one upon the others".

How did the Bourbaki conceived mathematics and, consequently, how did they study its history? In a paper of 1948, called "L'architecture des mathématiques", Bourbaki explained his conception, enlightening quite well the points which they chose to write the history and what sense has the modernity narrated in their book of 1960. A conception and a narrative which happened to be tremendously influential and have defined the outlines, the norms, priorities of what became the dominant history, the matrix and reference for generations of mathematicians and historians of modern mathematics.

They wrote, quotations are on this slide, that "The internal evolution of the mathematical science tightened, more than ever, the unity of its different parts […]. It can be summarized in a tendency that is the axiomatic method, [...] "which is performed thanks to the notion of structure which takes into account relations between elements without considering their own nature." Mathematics are therefore a container of abstract forms. Taking this process in consideration - a process which began at the end of the 19^{th} century – they brought the idea, I quote once more, that "connections between experimental world and mathematical one are accidental contacts".

This very conception of mathematics and their evolution since the 1870s built basics of the narratives written up to the 1980s, delineating this so called "royal road to me". This conception enjoined unhistoricised, and unquestioned evidences, enjoined Modernity universals which are: the ethos of pure mathematics; the triumph of abstraction, axiomatisation ; the growing autonomy of mathematics within outward reference.

II. The decades 1990-2000 : A first new agenda, but the same characters.

In the 1980s, the historians of modern mathematics are no more themselves practising mathematicians, they get, as historians of mathematics, an institutional identity and a greater autonomy with regard to mathematicians. They begin to appropriate and make use of new conceptual tools issued from other horizons than those of history of mathematics. They take **off** history of science, **off** history, the notions of « schools », « networks », « mathematical tradition » or « culture », "images" and "representations", which they use as analytical categories - loosely or with a more elaborate sense - to address the internal dynamics of a mathematical community. They address the way that this dynamics shape the "local" aspects of the mathematics produced by the community, "local" aspects as opposed to "universal". Therefore, they initiate a historical research which oversteps the bourbakiste's agenda and its unquestioned and constitutive universality.

This new understanding of historical investigation introduces and emphasizes components of mathematical knowledge linked to educational and institutional issues which were totally illegitimate in the previous bourbakist history. Mathematics studied with this new agenda are established into the realities of mathematical practices.

But I want to pay attention to another characteristic of this new agenda. Of which mathematical topics and practices do these historians write the history? We have to note that, in spite of the unquestionable enlargement of the mathematical scene, their favourite research field remains the same as in the previous agenda, involving the same mathematical fields and actors. The main affair, the driving matter in this history is still the conquest of modernity, of axiomatisation, autonomy, formalism and abstraction.

Herbert Mehrtens's book, *Moderne. Sprache. Mathematik,* published in 1990, has been one of the first examples of this issues renewing. It deals with the modernist transformation of mathematics in the early 20th century, mostly in Germany, and its links with an underlying modernity of other parts of contemporary culture. Modernism, in this book, is due to an "avant garde" of the mathematical professional group who defined the real, the pure and the most progressive type of mathematics characterised once more by formalism and autonomy, particularly within physics. On the other hand, mathematicians who developed or promoted other types of mathematics, like application-oriented mathematicians, are qualified by Mehrtens as "counter-moderns": Involvement in applications or in teaching matter cannot indeed fully be identified as the modernist attitude as defined here.

Nearly twenty years later, J. Gray addresses again this same question in a book published in 2008 entitled *Plato's Ghost : the modernist transformation of mathematics,* devoted to the decades 1890-1930. Gray investigates numerous mathematical fields developed in different national or local contexts that he characterises. He describes and analyzes what he considers as a decisive transformation of mathematical ontology, a single cultural shift (characterized by autonomy and emphasis on the formal aspect) which raised new standards, new foundations, new objects, I mean the "modernist ones".
Such a history, as Gray admits, is a partial one with inevitable distortion; but he does argue that it is the narrative of a coherent group of people who have built modern mathematics, mathematics of the 20^{e} century, our mathematics. However, this point of view, as I want to point out, is a quite excluding one.

First, it excludes important parts of mathematics of this period, new modernist objects being of interest to pure mathematicians almost exclusively, Gray says.

It excludes also actors. In his book Gray focuses on a quite small group of mathematicians whom he introduces according to their places and their mathematical domains. If I look at the French case, the case I know the best, who are those "French modernists" as Gray names them? They are "ten or so" at the turn of the 20th century. Ten among how many? A question which does not seem to make sense in Gray's perspective, but a question which can nevertheless be asked. For, between 1870 and 1914, the *Bulletin* of the French mathematical Society published articles written by 160 members.

This drastic selection which focuses on a few people, works also on an other scale. It is at the very scale of persons that Gray applies this selective bias, cutting into their mathematical production and activity to enrol them more handily in his narrative. I will exemplify this "modernist amputation", I dare say, with the case of one of them, the so "labelled" "French modernist" Emile Borel.

Borel was one of the rising stars of French Mathematics at the turn of the 20th century. He is presented by Gray as a creator of modernist objects in a field of obviously pure mathematics, the theory of functions and the measure theory of sets. So, he is called "abstract", is sided with "modernists" who left "concrete" applications to others, facing an older generation who felt that the new generality Borel and others introduced, was artificial, with an excessive formalism and degree of abstraction.

From this standpoint, according to the standards of mathematical modernity, Borel would have considered mathematics as an autonomous body of ideas with little or no outward reference. Yet, already in his thesis, where he defines these modernist objects, Borel shows an explicit care for applications of his results to physics; even more, a few years later, he writes that mathematics have to be the servant of physics and argues that the study of physics theories prevents from a too great inclination towards abstraction. This is central in Borel's scientific personality and would categorize him as a "counter-modern", yet Gray missed it.

Therefore, this history of mathematical modernity, necessarily written by excluding or amputating processes, is accompanied by the creation of manifest historiographical peripheries which can nevertheless be in the centre of the mathematical activity at that time , as shown by Borel's case.

III. An other new agenda : make the peripheral mainstream

During these last two decades, some historians of mathematics shifted the focus of their investigation towards the mathematical production published in the various periodicals of the early 20th century as listed in the bibliographical tools at that time.

The exploitation of these sources turns out to be more and more valuable owing to their digitalisation which allows interrogations of articles and authors on-line database, the achievement of networks of authors, quotations or references. It provides new tools which allow to embrace the scope of mathematical activities and of their protagonists, as they might have spread out according to the actors' inventories.

Different kinds of mathematical peripheries have thus been identified by historians.
Works on national mathematical communities, identified according to their membership to national mathematical societies which were set up in this period all over Europe and USA, have let into the mathematical scene a crowd of new figures, new actors. Studying their professions, their training, their activities, their productions, modified the perimeter and the configuration of the mathematical scene of the beginning of the 20th century as described in the dominating historiography. Entire mathematical domains come out, among which applied mathematics domains, as well as new places, journals or institutions connected at the time, in some way, to mathematical activity.

On an other scale, history of specific mathematical fields was not only improved but deeply modified, taking into account new actors revealed by those bibliographical and analytical tools which enable a systematic treatment of large corpus of articles. I'll take "modern" number theory as an example. Several historians of mathematics have shown how reduced is the history built from the first decades of the 20th century on a few exceptional mathematicians, first and for all Germans. Contrary to narratives delivered by collective memories, we have indications of a real activity in Germany but also in

France, Italy, Russia which was based on different networks. Only a part of one of these networks was appropriated by the traditional history of the field, the one which fits with "this royal road to me". Mainstream autonomous developments, unexpected connexions or solidarities between different mathematical communities had remained either invisible to the historians, or analyzed as a manifestation of mathematical decline, or at best delay.

Increasing once more the focus, historians have developped in the same way the analysis at the monographic scale. Specific places or institutions, specific individuals are then studied without any a priori cut.

But in what respect, and how, does this enlargement of the historical investigation to actors, domains, places, type of activities, until now ignored or marginalized to the mathematical periphery, have anything to do with the narrative of modernity?

I shall briefly answer, giving a couple of examples.

First, I'll take the example of the resurrection of a hitherto rather unknown French mathematician, Joseph Boussinesq, rediscovered because of his studies in the 1880s on fluid dynamics. So called "Boussinesq' equations " and some of his procedures for approximations have had a renaissance of interest being now used in fluid mechanic and in numerical analysis. So, mathematical actuality may force to reexamine mathematical modernity which may lies where it was not looked for. One "royal road" may hide another, or many others. The matrix of 20th century mathematics, of "our" mathematics does not lie in the sole pure and so called modern mathematics and its history cannot be narrated according only to what has been described as "modernist" trajectory.

Taking fields of applied mathematics into consideration has lead to challenge the modernist trajectory in another way and examine further the criteria of autonomy of mathematical knowledge and its objects, the criteria of abstraction and formalism. Historians of mathematics have shown that there were "modernist" conceptions of applied mathematics as well as shades of modernism oriented toward applications.

Borel's case, I have already mentioned, will allow me to enlarge the issues at stake with this reference to modernity (and its actors) in this history of mathematics. Obviously, Borel's sense of mathematical modernity is not exactly the same as, let us say, the historians Mehrtens or Gray. What was it at the early 20th century, understood as an actor's category?

It was various, depending on who was speaking and that is a first point.

But following Borel, it could be, and that is the great difference within historiographic modernist view, a conception which refers to the role mathematics could play within the modernity of the society and the time he lives in.

The scope of this modernity could be quite large. Scientific and technical progress, first: as a mathematician, Borel was interested in statistic mechanic, molecular theories, relativity; more, he promoted aviation and wrote mathematical book on this topic.

But also educational progress: as a mathematician, too, he was engaged in an educational reform of the French elite, claiming for introducing more life and more sense of reality in mathematics teaching, justifing this statement by the need to make the pupils realizing by themselves that Mathematics are not pure abstraction.

Last, social, economical modernity : Borel has done and written a lot in popularisation of mathematics to show the role they play in modern life.